Name:	
Class:	

JAMES RUSE A H S

GESTA NON VERBA

YEAR 12
ASSESSMENT TEST 1
TERM 4, 2013

MATHEMATICS EXTENSION 2

Time Allowed – 90 Minutes
(Plus 5 minutes Reading Time)

- *All* questions may be attempted
- *All* questions are of equal value
- Department of Education approved calculators and templates are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labeled Question 1, Question 2, etc.
Each question must show (in the top right hand corner) your Candidate Number.

QUESTION 1 (15 Marks) **Marks**

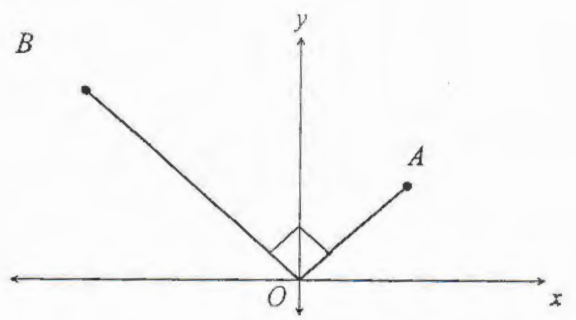
(a) Given that z and w represent the complex numbers $\sqrt{3}+i$ and $1-\sqrt{2}i$ respectively, find:

(i) $\frac{z}{w}$ in the form $x+iy$ **2**

(ii) the modulus and the argument of z **2**

(iii) the complex number C in $x+iy$ form, given that $\arg C = 2\arg z$ and $|C| = 3|z|$. **3**

(b)



On the Argand diagram, OA represents the complex number $z_1 = x + iy$, $\angle AOB = \frac{\pi}{2}$ and the length of OB is twice that of OA.

(i) Find the complex number represented by OB **2**

(ii) Given that AOBC is a rectangle, find the complex number represented by OC. **1**

(iii) Find the complex number represented by BA. **1**

(c) (i) If $(x+iy)^2 = -3-4i$, find all possible values of x and y **2**

(ii) Hence solve the equation $z^2 - 5z + (7+i) = 0$ **2**

QUESTION 2 (15 Marks) **Start a New page** **Marks**

(a) Indicate the region on the Argand diagram where the following inequalities hold simultaneously **3**

$$2 \le |z| \le 4 \quad \text{and} \quad \frac{\pi}{6} \le \arg z \le \frac{3\pi}{4}$$

(b) (i) Show that $\dfrac{(1+i)^8}{(1-\sqrt{3}i)^k} = 2^{4-k}\left[\cos\left(\dfrac{k\pi}{3}\right) + i\sin\left(\dfrac{k\pi}{3}\right)\right]$ **3**

(ii) For what values of k is $\dfrac{(1+\mathrm{i})^8}{(1-\sqrt{3}\mathrm{i})^k}$ purely imaginary? **2**

(c) Find the Cartesian equation for the locus of the point z if $\mathrm{Re}(z - \dfrac{1}{z}) = 0$ **3**

(d) It is known that 5+6i is a zero of the polynomial

$$P(x) = 2x^3 - 19x^2 + 112x + d \quad \text{where d is real}$$

(i) What are the other two zeroes of P(x)? **2**

(ii) What is the value of d? **2**

QUESTION 3 (15 Marks) **Start a new page** **Marks**

(a) Z lies on the locus defined by $|z+2| = 2$, let $\arg z = \theta$

(i) By use of an appropriate diagram, show that $\arg(z+2) = 2\theta - \pi$ **2**

(ii) Hence, or otherwise, find $\arg(z^2 + 6z + 8)$ **2**

(b) The complex number z=x+*i*y where x and y are real such that $|z - \mathrm{i}| = \mathrm{Im}(z)$

(i) Show that the locus of the point representing z has Cartesian equation $y = \dfrac{1}{2}(x^2+1)$. Sketch this locus **2**

(ii) Find the gradients of the tangents to this curve which pass through the origin. Hence find the set of possible values of the principal argument of z ($-\pi \le \arg z \le \pi$) **3**

QUESTION 3 (continued)

(c) (i) If w is one of the complex cube roots of unity, prove that the other complex root is w^2 **2**

(ii) Show that $1+w+w^2=0$ **1**

(iii) Prove that if n is a positive integer, then $1+w^n+w^{2n}=3$ or 0, depending on whether n is or is not a multiple of 3 **3**

QUESTION 4 (15 Marks) **Start a new page** **Marks**

(a) The roots of the equation $z^2-6iz+3=0$ are α and β
Without solving this equation. Prove that $|\alpha|+|\beta|\geq 6$ **2**

(b) (i) Use DeMoivre's Theorem to show that

$$(1+i\tan\theta)^n+(1-i\tan\theta)^n=\frac{2\cos n\theta}{\cos^n\theta}$$ where n is an integer ($\cos\theta\neq 0$) **2**

(ii) Use this result to show that the equation

$(1+z)^4+(1-z)^4=0$ has roots of $\pm i\tan\frac{\pi}{8}, \pm i\tan\frac{3\pi}{8}$ **3**

(iii) Hence or otherwise ,show that $\tan^2\frac{\pi}{8}=3-2\sqrt{2}$ **2**

(c) (i) Find in modulus argument form the roots of the equation $z^{2n+1}=1$ **2**

(ii) Hence factorise $z^{2n}+z^{2n-1}+......+z^2+z+1$ into quadratic factors with real coefficients **2**

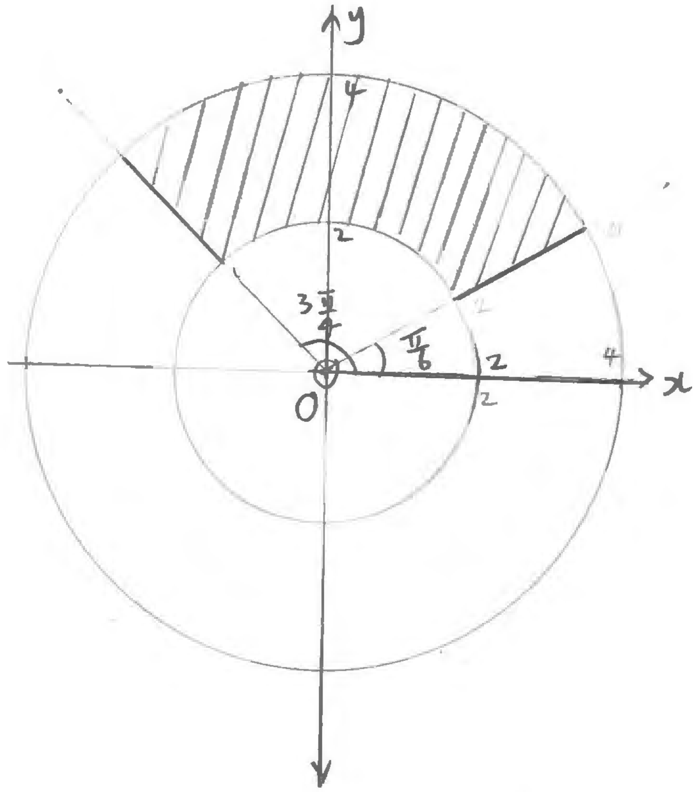
(iii) Deduce that $2^n\sin\frac{\pi}{2n+1}\sin\frac{2\pi}{2n+1}........\sin\frac{n\pi}{2n+1}=\sqrt{2n+1}$ **2**

END OF EXAM

1	Marks	Comments
(a) (i) $\dfrac{\sqrt{3}+i}{1-\sqrt{2}i}\times\dfrac{1+\sqrt{2}i}{1+\sqrt{2}i}$ $=\dfrac{\sqrt{3}+i+\sqrt{6}i-\sqrt{2}}{1+2}$ $=\dfrac{\sqrt{3}-\sqrt{2}}{3}+\dfrac{\sqrt{6}+1}{3}i$	1 1	
(ii) $\lvert z\rvert=\sqrt{3+1}$ =2 Arg z=$\tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ $=\dfrac{\pi}{6}$	1 1	
(iii) $\lvert C\rvert=6$ Arg C=$2\times\dfrac{\pi}{6}$ C=$6\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)$ $=6\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)$ $=3+3\sqrt{3}i$	1 1 1	
(b) (i) $OB=iOA\times 2$ $=i(x+iy)\times 2$ $=-2y+2ix$ (ii) $OC=OB+OA$ $=(-2y+2ix)+(x+iy)$ $=(x-2y)+(2x+y)i$ (iii) $BA=BO+OA$ $=-OB+OA$ $=-(-2y+2ix)+(x+iy)$ $=(x+2y)+(y-2x)i$	2 1 1	1 for i, 1 for 2

(c) (i) $x^2 - y^2 + 2ixy = -3 - 4i$ Equating terms $x^2 - y^2 = -3$ ....(1) $2xy = -4$ ....(2) $x^2 - \left(\frac{-2}{x}\right)^2 = -3$ $x^4 + 3x^2 - 4 = 0$ $(x^2 + 4)(x^2 - 1) = 0$ $x^2 = 1$ *or* $x^2 = -4$ $\therefore$ x = ±1 or no solution When x=1 ,y=-2,when x=-1, y=2	1 1	
(ii) $z = \frac{5 \pm \sqrt{25 - 4(7+i)}}{2}$ $z = \frac{5 \pm \sqrt{-3-4i}}{2}$ $z = \frac{5 \pm (1-2i)}{2}$ $z = \frac{6-2i}{2}$ or $\frac{4+2i}{2}$ z=3-i or 2+i	1 1	

2a)



1m for circles
1m for arg
1m for shading

bi) $1+i=\sqrt{2}\,\text{cis}\,\frac{\pi}{4}$, $1-\sqrt{3}i = 2\,\text{cis}\left(-\frac{\pi}{3}\right)$ — $\frac{1}{2}$m + $\frac{1}{2}$m

$$\frac{\left(\sqrt{2}\,\text{cis}\,\frac{\pi}{4}\right)^8}{\left(2\,\text{cis}\,-\frac{\pi}{3}\right)^k} = \frac{2^4\,\text{cis}\left(\frac{8}{4}\pi\right)}{2^k\,\text{cis}\left(-\frac{k\pi}{3}\right)} \quad \text{(De Moivre's Th)}$$

$\frac{1}{2}$m + $\frac{1}{2}$m — forgot D.M. Th $-\frac{1}{2}$m

$$= 2^{4-k}\,\frac{\text{cis}\,2\pi}{\text{cis}\left(\frac{k\pi}{3}\right)} = 2^{4-k}\,\text{cis}\left(2\pi - -\frac{k\pi}{3}\right)$$

$\frac{1}{2}$m (many forgot '$--$' $-\frac{1}{2}$m)

$$= 2^{4-k}\,\text{cis}\left(2\pi + \frac{k\pi}{3}\right) = 2^{4-k}\,\text{cis}\left(\frac{k\pi}{3}\right)$$

$\frac{1}{2}$m for $2^{4-k}\,\text{cis}\left(2\pi + \frac{k\pi}{3}\right)$

$$= 2^{4-k}\left(\cos\frac{k\pi}{3} + i\sin\frac{k\pi}{3}\right)$$

or

$$\frac{2^{4-k}\,\text{cis}\,2\pi}{\text{cis}\left(-\frac{k\pi}{3}\right)} = \frac{2^{4-k} \times 1}{\text{cis}\left(-\frac{k\pi}{3}\right)}$$

$\frac{1}{2}$m

$$= 2^{4-k} \cdot \left(\text{cis}\,-\frac{k\pi}{3}\right)^{-1}$$

$\frac{1}{2}$m

$$= 2^{4-k}\,\text{cis}\,\frac{k\pi}{3}$$

Q 2 b. cont'd

P2

ii) $\frac{(1+i)^8}{(1-\sqrt{3}i)^k}$ is purely imaginary if

$Re\left(\frac{(1+i)^8}{(1-\sqrt{3}i)^k}\right)=0$

i.e. $\cos\frac{k\pi}{3}=0$

$\frac{k\pi}{3}=\pm\frac{\pi}{2}+2n\pi \quad n\in J$ (forgot $n\in J$ − ½ m) 1 m

$K = 6n \pm \frac{3}{2}$ # 1 m

or

$\cos\frac{K\pi}{3} = \cos\left(\pm\frac{\pi}{2}+n\pi\right)$

$\frac{k\cancel{\pi}}{3}=\pm\frac{\cancel{\pi}}{2}+n\cancel{\pi}$

$K=\pm\frac{3}{2}+3n$ #

2c) $z-\frac{1}{z} = x+yi-\frac{1}{x+yi} \quad (z\neq0)$

$= x+yi-\frac{1}{x+yi}\times\frac{(x-yi)}{(x-yi)}$

$= x+yi - \frac{x-yi}{x^2+y^2}$ 1 m

$Re\left(z-\frac{1}{z}\right)=0$ if $x-\frac{x}{x^2+y^2}=0$ or equivalent 1 m

$x\left[1-\frac{1}{x^2+y^2}\right]=0$

$\therefore x=0$ or $\frac{1}{x^2+y^2}=1$

$(x^2+y^2=1)$

$\therefore$ Locus is $x=0$ or $x^2+y^2=1$ 1 m

exclude $(0,0)$

many students write $x\neq0$ (0 m)

2 d i) Since all coff of $P(x)$ are real | 1m

$\therefore 5-6i$ is also a root.

Let α be the third root

(Sum of Rts) $5+6i+5-6i+\alpha = \frac{19}{2}$

$\therefore \alpha = -\frac{1}{2}$

So zeros are $5+6i$, $5-6i$ or $-\frac{1}{2}$ # | 1m

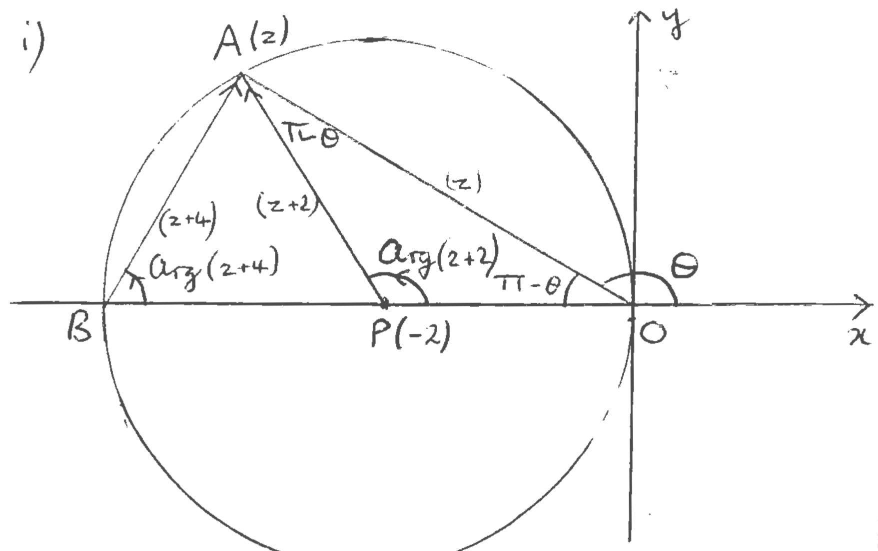
(ii) Product of Rts $= -\frac{d}{2}$

$(5+6i)(5-6i)(-\frac{1}{2}) = -\frac{d}{2}$ | 1m

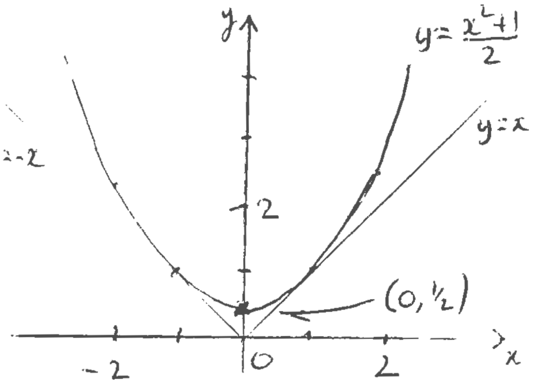
$\therefore d = 25+36$

$\underline{\underline{d = 61}}$ | 1m

TERM 4 2013 YR 11 X2 **MATHEMATICS: Question** 3 p1 of 3

Suggested Solutions	Marks	Marker's Comments
a) i) 		1 mark for a well labelled diagram
Let the point A represent z, P represents −2. $\angle AOP = \pi - \theta$ (Straight angle adds to π) $AP = PO = 2$ (Radii of circle) $\therefore \angle PAO = \angle AOP = \pi - \theta$ (Equal angles opposite equal sides in ΔAOP) $\angle APO = \pi - 2(\pi - \theta)$ (Angles in ΔAOP add to π) $= 2\theta - \pi$ But $\angle APO = \text{Arg}(z+2)$ $\therefore \text{Arg}(z+2) = 2\theta - \pi$	2	Given result. Take care to provide documented logic trail.
ii) $\text{Arg}(z^2+6z+8) = \text{Arg}(z+4)(z+2)$ $= \text{Arg}(z+4) + \text{Arg}(z+2)$ $= \angle PBA + (2\theta - \pi)$	1	
Since $\angle BAO = \pi/2$ (Angle in semicircle) $\angle PBA = \pi - (\pi/2 + \pi - \theta)$ (Angles in ΔABO) $= \theta - \pi/2$ $\therefore \arg(z^2+6z+8) = \theta - \frac{\pi}{2} + 2\theta - \pi = 3\theta - \frac{3\pi}{2}$	1	

MATHEMATICS: Question 3 p2 of 3

Suggested Solutions	Marks	Marker's Comments
b) i) $z = x + iy$ $\quad x, y$ real		
$\lvert z - i \rvert = \operatorname{Im}(z)$		
$\lvert x + i(y-1) \rvert = y$		
$\therefore x^2 + (y-1)^2 = y^2$		
$x^2 + y^2 - 2y + 1 = y^2$		
$2y = x^2 + 1$		
$\underline{\underline{y = \frac{1}{2}(x^2+1)}}$	1	
	1	The diagram needed to show $(0, \frac{1}{2})$ as the vertex in some way. It should be an easy mark.
ii) Let the tangent through the origin be $y = mx$.		There was some awful garbage presented here.
Solve against $y = (x^2+1)/2$		
$x^2 + 1 = 2mx$		
$0 = x^2 - 2mx + 1$		
$\Delta = 0$ for tangent $\Rightarrow (2m)^2 - 4 = 0$		
$\therefore \underline{\underline{m = \pm 1}}$	1	
All the points z lie between these two tangents which are at angles $\pi/4$ and $3\pi/4$	1	
$\therefore \underline{\underline{\frac{\pi}{4} \leq \operatorname{Arg}(z) \leq \frac{3\pi}{4}}}$	1	½ off for people who got order wrong way.

MATHEMATICS: Question 3		p3 of 3
Suggested Solutions	**Marks**	**Marker's Comments**
c) i) $\omega^3 = 1$ (Given – ω is a cube root of unity) $(\omega^2)^3 = \omega^6$ $= (\omega^3)^2$ $\therefore$ ω^2 is also a root $= 1^2$ (but is it different) $= 1$	1	
Assume that $\omega^2 = \omega$ Then $\omega(\omega - 1) = 0$ $\therefore \omega = 0$ or 1 # since ω is non real.	1	
$\therefore$ ω^2 is a different non real root.		
ii) $1, \omega, \omega^2$ are the three roots of $z^3 - 1 = 0$. Sum of roots $=$ "$-b/a$" $= 0$ $\therefore 1 + \omega + \omega^2 = 0$	1	If $(\omega-1)(\omega^2+\omega+1)=0$ used then reason should be given why $(\omega - 1) \neq 0$.
iii) Three cases: n divisible by 3 $n = 3k$ $k \in \mathbb{Z}$ $1 + \omega^n + \omega^{2n} = 1 + \omega^{3k} + \omega^{6k}$ $= 1 + (\omega^3)^k + (\omega^3)^{2k}$ $= 1 + 1^k + 1^{2k}$ $= 1 + 1 + 1 = 3$	1	Lot of waffly answers using patterns etc. Marks were given pro rata.
If not divisibly by 3 (a) $n = 3k+1$ $1 + \omega^n + \omega^{2n} = 1 + \omega^{3k}\omega + \omega^{6k}\omega^2$ $= 1 + \omega + \omega^2$ $= 0$ (from part ii)	1	
b) If $n = 3k+2$ $1 + \omega^n + \omega^{2n} = 1 + \omega^{3k}\omega^2 + \omega^{6k}\omega^4$ $= 1 + \omega^2 + \omega^4 = 1 + \omega^2 + \omega$ $= 0$	1	

Conclusion

MATHEMATICS Extension 2: Question 4

Suggested Solutions	Marks	Marker's Comments
(a) $z^2 - 6iz + 3 = 0$		
sum of roots: $\alpha+\beta = -b/a$		
$= -(-6i)$		
$= 6i$	1 mk.	
$\lvert\alpha\rvert + \lvert\beta\rvert \geq \lvert\alpha+\beta\rvert$ by the triangle inequality		$-\frac{1}{2}$ mk off if you didn't mention the triangle inequality
$\therefore \lvert\alpha\rvert + \lvert\beta\rvert \geq 6$	1	
(b)(i) LHS $= (1+i\tan\theta)^n + (1-i\tan\theta)^n$		
$= \left(\frac{\cos\theta}{\cos\theta} + \frac{i\sin\theta}{\cos\theta}\right)^n + \left(\frac{\cos\theta}{\cos\theta} - \frac{i\sin\theta}{\cos\theta}\right)^n$		
$= \frac{\cos n\theta + i\sin n\theta}{\cos^n\theta} + \frac{\cos n\theta - i\sin n\theta}{\cos^n\theta}$		by De Moivre's Theorem 1 mk.
$= \frac{\cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta}{\cos^n\theta}$		
$= \frac{2\cos n\theta}{\cos^n\theta}$	1 mk.	
= RHS		
(ii) $(1+z)^4 + (1-z)^4 = 0$		
let $z = i\tan\theta$		
$(1+i\tan\theta)^4 + (1-i\tan\theta)^4 = 0$		
$\frac{2\cos 4\theta}{\cos^4\theta} = 0$ from (i)	1 mk	
$\therefore 2\cos 4\theta = 0$		
$\cos 4\theta = 0$		
* Trig equation so it could be done using general solutions but most forgot the formula! $4\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$ (as it's a quartic and so only has 4 solns)		
$\theta = \pm\frac{\pi}{8}, \pm\frac{3\pi}{8}$	1 mk	
now $z = i\tan\theta$		
$\therefore z = i\tan\frac{\pi}{8}, i\tan -\frac{\pi}{8}, i\tan\frac{3\pi}{8}$		$i\tan -\frac{3\pi}{8}$

$\therefore z = i\tan\frac{\pi}{8}, -\tan\frac{\pi}{8}, i\tan\frac{3\pi}{8}, -i\tan\frac{3\pi}{8}$ (as $\tan -\theta = -\tan\theta$)

* max. 2 mks
* most students didn't explain why they had 4 solutions (1 mk) they appear to just "copy" what was given in the question!

MATHEMATICS Extension 2: Question 4

Suggested Solutions	Marks	Marker's Comments
(b)(iii) $(1+z)^4 + (1-z)^4 = 0$		
$1+4z+6z^2+4z^3+z^4+1-4z+6z^2-4z^3+z^4=0$		
$2+12z^2+2z^4=0$		
$1+6z^2+z^4=0$	1 mk	
$z^2 = \frac{-6 \pm \sqrt{32}}{2}$ Quadratic formula		
$= -3 \pm \sqrt{8}$		
$= -3 \pm 2\sqrt{2}$	½ mk	
now $z = i\tan\frac{\pi}{8}$		
$\left(i\tan\frac{\pi}{8}\right)^2 = -3 \pm 2\sqrt{2}$		
$-1\tan^2\frac{\pi}{8} = -3 \pm 2\sqrt{2}$		
$\tan^2\frac{\pi}{8} = 3 \mp 2\sqrt{2}$		
now $0 < \tan\frac{\pi}{8} < 1$ (as $\tan\frac{\pi}{4} = 1$)		$\frac{1}{2}$ mk.
$\therefore 0 < \tan^2\frac{\pi}{8} < 1$		
$\therefore \tan^2\frac{\pi}{8} = 3 - 2\sqrt{2}$ only		
* needed to justify properly why it was $3-2\sqrt{2}$, a lot of students fudged this!!!		

$3/4$

MATHEMATICS Extension 2: Question ~~…~~ 4

Suggested Solutions	Marks	Marker's Comment

Question 4

$z^{2n+1} - 1 = 0$

(c) (i) $z^{2n+1} = 1$

let $z = \operatorname{cis}\theta$

$(\operatorname{cis}\theta)^{2n+1} = 1$

(by De Moivre's)

$\operatorname{cis}(2n+1)\theta = \operatorname{cis}(0 + 2k\pi)$ where **1**

$k \in \mathbb{Z}$

$(2n+1)\theta = 2k\pi$

$\therefore \theta = \dfrac{2k\pi}{2n+1}$ for $k = 0, 1, 2, \dots, 2n$

or $k = 0, \pm 1, \pm 2, \dots \pm n$

$\therefore z = \operatorname{cis} 0, \operatorname{cis}\dfrac{2\pi}{2n+1}, \operatorname{cis}\dfrac{4\pi}{2n+1}, \dots \operatorname{cis}\dfrac{4n\pi}{2n+1}$ **1**

or $\operatorname{cis} 0, \operatorname{cis}\dfrac{2\pi}{2n+1}, \operatorname{cis}\dfrac{-2\pi}{2n+1}$

(ii) $z^{2n+1} - 1 = (z-1)(z^{2n} + z^{2n-1} + \dots z^2 + z + 1)$

so

$z^{2n} + z^{2n-1} + \dots z^2 + z + 1$

$= \cancel{\left(z - \operatorname{cis}\dfrac{2\pi}{2n+1}\right)\left(z - \operatorname{cis}\dfrac{4n\pi}{2n+1}\right) \dots \left(z - \operatorname{cis}\dfrac{\pi}{2n+1}\right)}$ **1**

$\left(z - \operatorname{cis}\dfrac{2\pi}{2n+1}\right)\left(z - \operatorname{cis}\left(\dfrac{-2\pi}{2n+1}\right)\right) \dots \left(z - \operatorname{cis}\dfrac{2n\pi}{2n+1}\right)$

~~Equating~~ Grouping conjugate pairs

$\left(z - \operatorname{cis}\dfrac{2\pi}{2n+1}\right)\left(z - \overline{\operatorname{cis}\dfrac{2\pi}{2n+1}}\right) \dots \left(z - \operatorname{cis}\dfrac{2n\pi}{2n+1}\right)\left(z - \overline{\operatorname{cis}\dfrac{2n\pi}{2n+1}}\right)$

$\cancel{\operatorname{cis}\dfrac{2\pi}{2n+1} = \operatorname{cis}\dfrac{2n\pi}{2n+1}}$

Marker's Comment:

Students using $\dfrac{2n\pi}{2n+1}$ got 0.

Needed to indicate end points ($\pm 1, \pm 2$) $k = 0, \dots$

Poorly done too many students did not indicate values for k.

don't forget

realise if [illegible]

$z^5 = 1 \quad z = \operatorname{cis}\dfrac{2k\pi}{2\times 2+1} \quad k = 0, \pm 1, \pm 2$

4/4

MATHEMATICS Extension 2: Question 4

Suggested Solutions	Marks	Marker's Comment
Question 4		
(c)		
$\therefore \left(z^2 - 2\left(\cos\frac{2\pi}{2n+1}\right)z+1\right)\left(z^2-2\left(\cos\frac{4\pi}{2n+1}\right)z+1\right)$		
$\cdots\left(z^2-\left(2\cos\frac{2n\pi}{2n+1}\right)z+1\right)$ — 1	1	
needed to [illegible]		
(iii)		
Now $z^{2n}+z^{2n-1}+\cdots+z^2+1=$		
$\left(z^2-2\cos\frac{2\pi}{2n+1}z+1\right)\cdots\left(z^2-\cos\frac{2n\pi}{2n+1}z+1\right)$		
Let $z=1$		
$1+1+\cdots+1+1$		
$=\left(1-2\cos\frac{2\pi}{2n+1}+1\right)\left(1-2\cos\frac{4\pi}{2n+1}+1\right)\cdots\left(\cancel{1}-2\cos\frac{2n\pi}{2n+1}z+1\right)$	1	
$\therefore 2n+1=2\left(1-\cos\frac{2\pi}{2n+1}\right)2\left(1-\cos\frac{4\pi}{2n+1}\right)\cdots 2\left(1-\cos\frac{2n\pi}{2n+1}\right)$		
Now $1-\cos 2\theta = 2\sin^2\theta$		needed to state this, accepted if [illegible] $(1-(1-\sin$ [illegible]
$2n+1=2^n\cdot\left(2\sin^2\frac{\pi}{2n+1}\cdot 2\sin^2\frac{2\pi}{2n+1}\cdots 2\sin^2\frac{n\pi}{2n+1}\right)$		
$2n+1=2^{2n}\left(\sin^2\frac{\pi}{2n+1}\sin^2\frac{2\pi}{2n+1}\cdots\sin^2\frac{n\pi}{2n+1}\right)$		problems with power of 2.
$\sqrt{2n+1}=2^n\sin\frac{\pi}{2n+1}\cdots\sin\frac{n\pi}{2n+1}$.	1	

1 if not [illegible] have.

J:\Maths\Suggested Mk solns template_V2.doc

Deduce ≠ Hence show